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Gaseous Particulate Interaction in a 3-Phase Granular Simulation

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ABSTRACT

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As computer generated special effects play an increasingly integral role in the development of films and other media, simulating granular material continues to be a challenging and resource intensive process. Solutions tend to be pieced together in order to address the complex and different behaviors of granular flow. As such, these solutions tend to be brittle, overly specific, and unnatural. With the introduction of a holistic 3-phase granular simulation, we can now create a reliable and adaptable granular simulation.

Our solution improves upon this hybrid solution by addressing the issue of particle flow and correcting interpenetration amongst particles while maintaining the efficiency of the overall simulation. We achieve this by projecting particles onto a 2D manifold and implementing density correction using a quadratic solver. Particle updates are projected back into 3D to spread the particles apart on each frame.

Keywords: granular simulation, unilateral incompressibility, compression

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Introduction

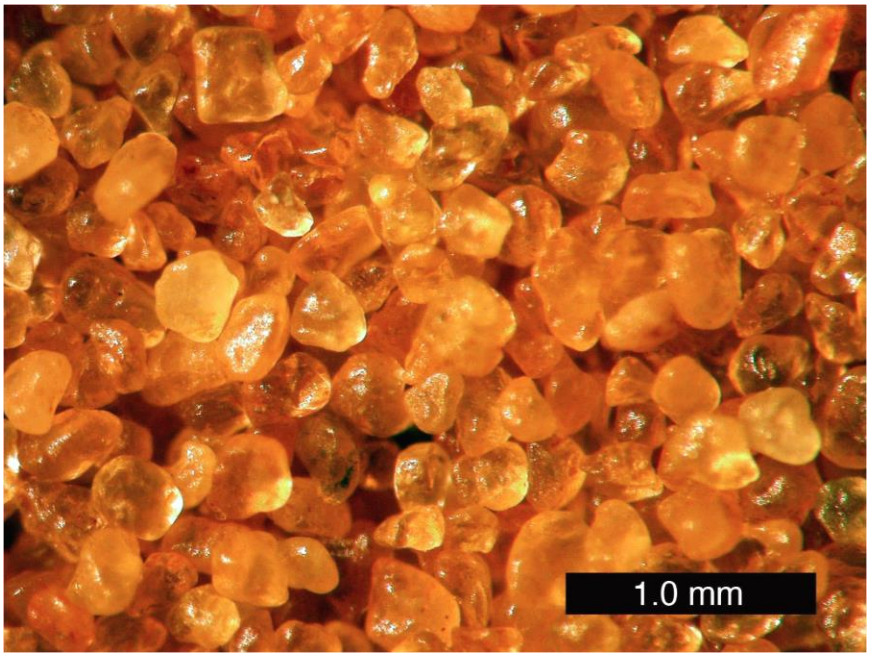


Image 1: Close-up of sand from the Coral Pink Sand Dunes State Park in Utah. These grains are rough and misshapen allowing for sand to interlock creating friction, which allows it to form a pile.

As computer-generated special effects become more prevalent in media, more challenging natural phenomena have been simulated and presented in a believable manner. Simulating fine particulate systems such as sand, soot, detergent and flour have historically been difficult as they exhibit a wide variety of behavior that mimic other phenomena. A pile of volcanic rock may act completely solid at times, then at other times exhibit fluid-like flowing with rockslides. Flour can pile as it is poured from a sack. Soot can cloud up if disturbed. Detergent can flow in interesting and unpredictable patterns as it clumps and breaks up.

Sand has received special attention as early as the 1980's. Its applications range from modeling natural phenomena such as weathering and erosion to entertainment. Sand can form piles. Granules can interlock to form micro arches. As sand avalanches certain granules can actually rise. As sand gets deeper, it doesn't increase in pressure, as opposed to water, which is why it can be used in an hourglass: its rate of flow is constant.

Understanding sand can be crucial in assessing the risk of debris flow, which puts certain populations around the world in danger. Sand's ability to allow fluid to percolate through it destabilizes its structure. It can lead to mud slides that flow like a liquid but behave like cement.

Image 2: Sinkhole in Guatemala.

Observing sand can lead to efficiencies in mineral displacement and energy exploration. Fracturing, or mining natural gas with the use of high-pressured water, is of great interest to our energy sector. Environmental hazards like sink holes reveal sand's interstitial structure and by understanding sand's behavior, we can understand the

consequences of various displacement techniques in use today. Understanding basic behaviors can give us insight in preservation of natural phenomena as well as understanding our geological history.

The simulation of granular materials can also be used in entertainment. This can add a layer of depth and realism to movies, enabling the audience to more fully experience emotions. In Pixar's Toy Story 3, our heroes find themselves helpless as they are drifting among debris in a garbage incinerator. The audience can relate to the emotion drawing upon their earlier experience as children in a ball pit. During the making of the film, a ball pit was set up in Pixar's main office for the artists and technicians to revisit and explore the emotion.

Image 3: Still of Woody in the garbage incinerator in Toy Story 3.

The complexity of sand can lead to the effect being avoided in animation all together make the case for our research into efficient sand simulation. The simulation of sand can also tie together computer-generated effects with their natural counterparts, though sand simulations are often left out because they slow production and raise costs. In Michael Bay's Transformers, for example, the U.S. military engaged in battle with a scorpion-like

robot known as Scorpnorok. After losing the engagement, Scorpnorok burrows into the ground for safety. The act of digging without a simulation resulted in the scorpion looking fake and floating as it dug down. It also meant that the camera angle needed to be horizontal with the ground to try to hide the character and ground interaction as much as possible to avoid the CGI popping effect as the visual accuracy of avalanching sand is hard to sell to an audience due to its complex behavior. A performant and visual plausible sand simulation is necessary to provide the industry with a wider toolbox to include sand in their media.

Image 4: Still of Scorpnorok burrowing into sand in Transformers.

The need for a holistic sand simulations is far reaching. Sand would take on a different purpose in the Avengers. Throughout the film, characters are constantly hitting and breaking buildings and streets. Concrete and asphalt fracturing exhibit behaviors of piling and avalanching. As rocky debris scatters these interactions are largely ignored. A cement barrier that fractures, piles and slides as it hits the street would not only make the effect more believable, but speed up production as artists no longer would have to manually simulate piling separately and thus drive special effects costs down.

Simulating millions of particles of sand is challenging. Each grain of sand has a unique material property and shape. Just a handful of sand contains hundreds of thousands of these particles, all of which collide against each other and interlock in different ways creating friction and stress while at times allowing movement and collision. These unique properties give sand the ability to act as a solid at times and flow like a fluid at others.

Holladay and Egbert have addressed the need to have an efficient and visually compelling granular simulation using a hybrid technique [3] (see image 5). Sand is comprised of three separate simulations including static geometry, a fluid solver featuring two-way coupling of friction and pressure, and free flowing particles. The efficiency gains of having sand that is further from the surface simplified is appealing as it allows artists to create and iterate granular simulations with greater speed and lower computational costs resulting in special effects studios saving money.

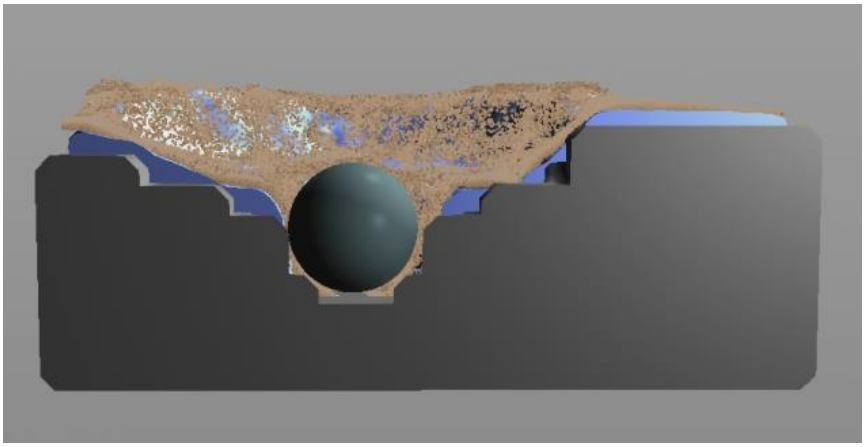


Image 5: Cutaway of Holladay and Egbert's hybrid simulation revealing the solid, liquid, and gas states.

Our solution builds upon a hybrid simulation of sand which seeks efficiency and visual plausibility over accuracy. Visual plausibility is necessary for effects to be believable to a film audience. Accuracy can be slow and costly and is often not necessary in a cinematic context. The hybrid technique relies on more complex states near the surface to maintain integrity of the system while keeping simulation frames down.

The most complex state of the hybrid solver is the gaseous state which consists of a thin layer of particles that glide on top of the fluid simulation below. This state uses simplified particle physics as physically-based simulation would introduce a computational bottle neck on the system. The lack of collision or particle-particle interaction on the surface can lead to undesirable artifacts that detract from the overall simulation. These special cases are similar to edge cases that appear in general simulation requiring the programmer to manually patch or deflect them. We propose a general modification to the gaseous state by introducing a simplified algorithm that addresses the issue when particles can clump up together and interpenetrate leading to visual flaws in the simulation.

We solve this by adding a density constraint into the system. This constraint is enforced with a density correction step that uses a quadratic minimization solver to produce a pressure gradient. Particles are displaced from high pressure into low pressure zones. A key contribution of our research involves flattening the particles from 3 dimensions into 2 dimensions to linearly reduce the problem space of our solver. This is possible by acknowledging the tendency of surface particles to flatten out. Particles that sink too far are converted into the separate liquid phase. The vertical variability of the particles are not a primary component in the distribution. Applying the density correction

step with reduced dimensions means clumping and interpenetrating particles are prevented in typically what is the most expensive simulation step without creating the computational bottleneck in other approaches.

The remainder of this thesis is organized as follows: Chapter 2 addresses previous work and methods for simulating sand efficiently. Chapter 3 reviews the hybrid solution introduced by Holladay and Egbert that we use as the basis for the work presented here. Chapter 4 describes our algorithm and how it fits in the context of the hybrid solution. Chapter 5 demonstrates results of the algorithm. Chapter 6 discusses future work. Chapter 7 provides some concluding remarks relative to this thesis.

Chapter 2

Related Work

Sand has been observed and modeled to highlight and simulate varying behaviors. Liu, Kaplan, and Gray first considered the fractal and recursive behavior of sand, observing how it can settle, form patterns, bridges and fractal structures that repeat on multiple scales [17]. Cundall and Strack first developed a discrete numeric model for particles to collide and were the first to discover the properties of granular assemblies [1]. McCarthy and Ottino recognized the system-wide behaviors of sand and modeled equations to capture avalanching, friction, and sliding of sand inside a tumbler [4]. Mehta et al. have revisited the piling and arching of sand producing a model of sand that takes into consideration the angle of repose and handles collisions through discrete element methods [20].

2.1 Discrete Element Methods

The most accurate and realistic looking approach to simulating granular phenomena is to use Discrete Element Methods (DEM). DEM takes an intuitive and straight-forward approach. Each particle is simulated as a rigid body. A simple piece of geometry stands in for the particle and simulates its 3-dimensional position, velocity, and momentum. Spheres are chosen as a proxy because the collision detection is efficient. A particle can be represented by a small group of spheres, known as an assembly (see image 6), of random sizes frozen together to mimic interlocking and friction on a particulate basis.

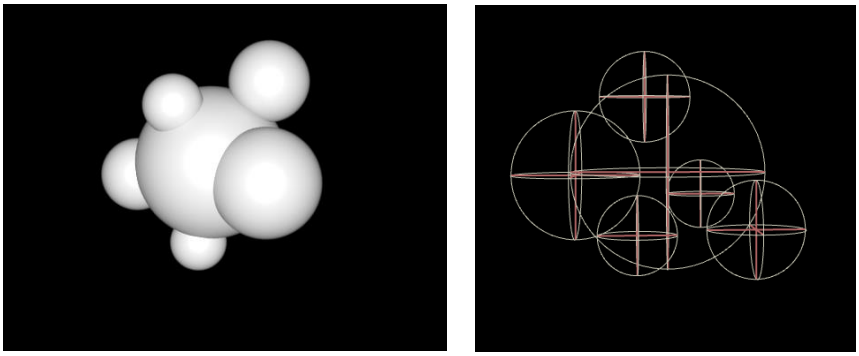


Image 6: Granular Assembly comprised of frozen spheres.

The DEM algorithm integrates all Newtonian forces and particles and updates their velocities and positions. This approach is computationally expensive and necessitates limitations on either the number of particles in the simulation or the number of frames simulated, though with some simulations multiple particles track a single near-by particle to reduce complexity. While DEM solutions work for scenes of small scale, they become prohibitively expensive as more particles are added.

The discrete element method has been improved upon by Fortin, Millet, and Saxce through projecting friction onto a cone when dealing with compaction [7]. Irving et al. revealed a superior method for resolving multiple contact points between non-convex bodies [11]. When spherical particulate assemblies are used in proxy of granules, they speed up computation. Particles can even be used as underlying rigid body collision proxies for any arbitrary geometry [15]. Coarse grain collision detection can be tuned when dealing with granular flow to find potential collision pairs faster [22]. Liu et al. have found that by using primary component analysis on the coarse axis sweep step, the complexity of finding potential collision pairs can be reduced [18]. When colliding thousands of particles, fine-grained steps can be culled from particles not currently acting upon geometry [19]. Our

simulation seeks to avoid paying the high computational cost of simulating individual granules and their collisions at an individual level. With our work addressing surface particles, DEM methods would be a computational bottleneck for the simulation, and therefore unsuitable.

2.2 Height Maps

In order to reduce the number of particles in a scene, height maps [31] are often used. Discrete 2-dimensional height maps are rectangular columns that stand as proxies for particles [32]. A pile of granules is mostly solid and therefore can be frozen, or temporarily removed from computation. Only the surface needs to be simulated to mimic granular phenomena. In a height map, a 3-dimensional surface is constructed using a 2-dimensional grid of height values that determine the surface of the solid mesh (see Image 7). The surface holds the remaining top layer of particles that are simulated with DEM. Height maps have been constructed together with DEM models so that they can adaptively grow and replace particles or shrink and produce additional particles.

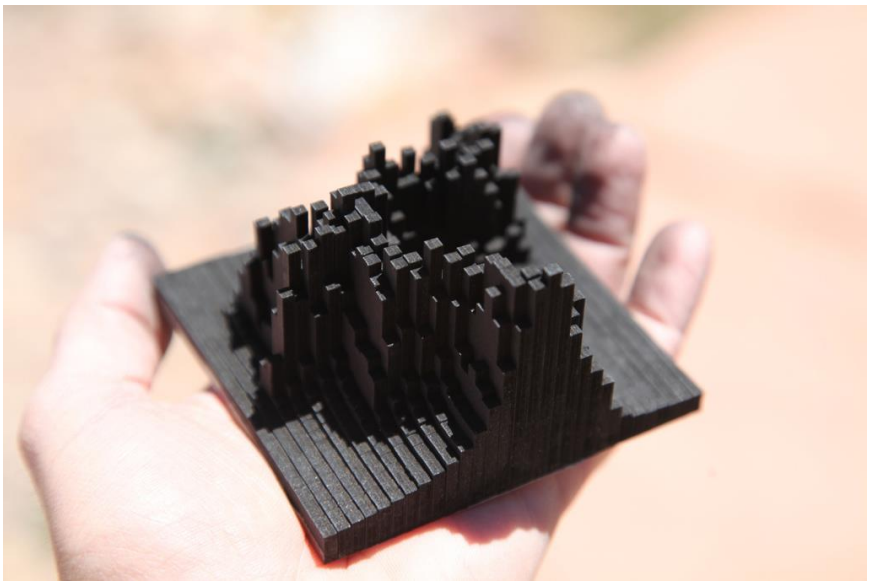


Image 7: A Height Map sliced by a laser engraver.

Although height maps provide some computational relief, they still suffer from having to provide a thick layer of particles sufficient to cover their surface if particulate surface detail or granular flow is required, which is computationally expensive. Particulate flow can mean a majority of the sand has to be solved using DEM while only a small portion can be culled out. Height maps also suffer from requiring sand to be on flat, horizontal surfaces [32]. This allows for effects such as foot prints and tracks on a beach [25], but does not scale well for simulating effects such as sand being poured from a bucket. Height maps benefit simulations with deep layers of granules. Our particle layer is a thin covering and thus would not be effective at reducing computation as the surface particles would still have to be simulated.

2.3 Proxy Methods

Other mesh-like algorithms can settle particles in place or replace still particles with proxy geometry to simplify the number of actors in a scene [33]. While these approaches solve some of the issues that come with height maps, they still suffer from having to simulate a large number of particles using DEM. These techniques have allowed detail to be allocated to where it is most important. Adams et al. have shown that particles can be adaptively down-sampled while preserving detail in the fluid simulation [16]. Zhang et al. authored a similar approach that also combines deep-water particles and is GPU friendly. Hsu and Keyser developed a technique in which as they piled rigid bodies, they would freeze pieces that have stopped moving and were below the surface to speed up computation [9]. Irving et al. revealed that by using a grid that was made of tall and thin cells, deep below the surface, they could use less cells but keep the horizontal grid resolution, which was responsible for surface wave effects [10]. Recently, Kim et al. have opted to upscale a fluid simulation by adding detail entirely to the surface wave simulation while on a coarse grid below [24]. Proxy methods could simplify the simulation of particles by freezing them onto control particles. Our method, however, seeks to avoid the interpenetration of particles which previous proxy method fails to address.

2.4 Sand as a Fluid

As sand avalanches and flows, it can be modeled similar to a liquid. Fluid solvers are able to efficiently simulate fluid behavior through essentially sub-sampling the phenomena. Fluids can be simulated with either grid-based Eulerian approaches or particle-based Lagrangian methods. A hybrid or Fluid Implicit Particle (flip) solver uses particles to track the fluid

while using a volumetric approach to calculate pressure [13]. As fluids are solved using pressures and their number of particles are drastically smaller than granules, they provide a great speed-up over DEM methods. Drawing inspiration from McCarthy and Ottino, Zhu and Bridson explore the continuum properties of sand by using fluid equations [2] that Bridson had worked on [14]. They modified an Eulerian grid to only animate cells where the sand is above the angle of repose and able to overcome frictional forces, otherwise making them solid.

Chang et al. improve on this approach using smooth particle hydrodynamics (SPH) allowing for better splash effects [5]. SPH is a Lagrangian approach to fluid solvers where fluid information is tracked on a particle bases. The movement of particles corresponds to the movement of the liquid. SPH was first introduced by Monaghan [13] and extended to account for multi-phase behavior [21]. Alduá and Otaduy also take an SPH approach eliminating grid-like artifacts from Zhu and Bridson's earlier work [8]. The SPH approach has also allowed for fluids and granules to be co-simulated [12]. A drawback of using SPH to simulate liquid in general is the instability of the system when particles get too close. Particles that end up too close together explode in all directions as stiff spring forces react adversely to small floating point values.

Fluid-based methods for simulating granular phenomena in general suffer from various issues. While granules can take on fluid-like characteristics when avalanching, they aren't fundamentally fluids. Granular phenomena simulated as a fluid looks and acts like quicksand or mud. Fluid based methods suffer from particles that separate from each other, which makes them look and behave like globs of mud. Fluids also pile fundamentally

different as they rely on surface tension instead of friction to hold their material together. Simulating surface particles as a fluid would result in less surface detail. Fluid simulation steps smooth over fine movement which would make the surface look artificial. Surface particles need to be able to break free from the surface when granules avalanche and splash. Simulating surface particles as a fluid also adds additional computational complexity. The hybrid solver uses surface particles to add detail. The current work has issues with clumping on the surface (Image 8). Our solution makes the granules appear less clumped. This will be covered in more detail in Chapter 4.

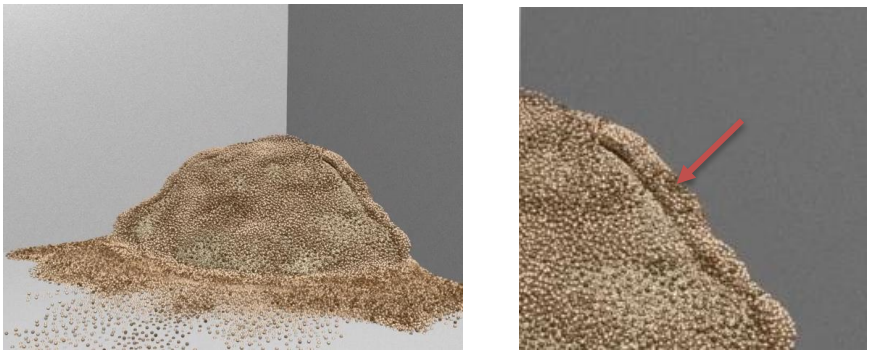


Image 8: Surface granules clumping together along an edge

DEM, height maps, and fluids all have major drawbacks that reduce the believability of the granular phenomena by either making them look less accurate or by reducing the number of particles as a trade-off to render time. Combining these different approaches can allow for some better trade-offs in appearance and complexity. Sand is described as three phases of matter and interstitial fluid is addressed by Forterre and Pouliquen [6]. Holladay and Egbert developed a hybrid system that allowed underlying particles to solidify and

liquefy depending on their motion and how close they are to the surface allowing for a fast and realistic simulation [3].

Discrete element methods are the most accurate approaches for simulating granular materials, but their complexity is undesirable for the rapid iteration needed for adding special effects in movies. Fluid approaches reduce complexity, but at the cost of realistic behavior. A hybrid approach using both fluid and DEM methods solves the trade-off between fluid and discrete element methods, but using discrete element methods still introduces too much complexity. Our approach seeks to replace the standard DEM method for particles.

All existing methods for simulating sand fall short. Discrete Element Methods are too computationally expensive. Fluids methods suffer smoothing effects. Proxy methods cannot be applied to the surface while maintaining detail. Our solution is to address these short-comings by applying a light-weight density correction step that pushes particles apart.

Chapter 3

Hybrid Granular Simulation

We review the Hybrid simulation by Holladay and Egbert as our research builds upon the hybrid solution. We propose an algorithm that preserves particulate detail on the surface of the simulation. We will give sand its characteristic look and feel by introducing particles that bounce, splash without clumping up or looking like a viscous fluid. In order to describe our solution, we must first describe the approach we have taken to simulate sand.

In the hybrid solution, granular phenomena can be characterized by 3 distinct phases: solid, liquid, and gas. Granules are solid-like in their lowest energy state. Microscopic particles are rough and interlock forming tight and incompressible structures. Sand can pile up and maintain a slope, something that is impossible in a liquid. As sand is poured, it will continue to form a bigger pile until the angle of the pile becomes steeper than the material's angle of repose. This angle describes the angle in which the forces of gravity overcome the forces of friction in the material leading to granules breaking off and sliding down the pile.

While in motion or avalanching, particles tend to stay in motion and mimic behaviors found in fluids, behaviors such as incompressibility, cohesion, and dispersion. Particles will flow until the friction of the material overcomes the momentum and gravitational forces acting on the particle. During particulate flow, particles may collide with one another and bounce into the air separating entirely from the system.

Airborne particles resemble gaseous particulate interactions. These particles are generally free from interaction with other particles until they either leave the system or fall back onto the material. In the latter case, they may either rebound or join the majority of the material. Airborne particles follow gaseous properties and can be modeled with simple Newtonian forces.

Holladay and Egbert's hybrid solution describes and implements their simulation in three states, which characterize the three main approaches to granular simulation. Particles that are below the surface of the simulation and are not currently in motion are culled out and are replaced by a proxy geometry in the form of a triangulated mesh whose collision can be proxied by freezing the particles that border its space. This geometry can grow as the pile gets deeper or more particles settle. Conversely, when the pile suddenly shifts or vibrates, an exterior object penetrates or other forces act on the pile, the solid geometry returns to its particulate state. As our research builds upon the gaseous state of the hybrid solution, we include relevant details of the hybrid solution in sections 3.1, 3.2, and 3.3 for completeness.

3.1 Solid

The solid geometry can be carved off and repopulated with granules in preparation for simulating moving particles [3]. The amount of geometry converted to the fluid state depends on the magnitude of the force acting upon it. Our simulation handles this situation by allowing a portion of the material to be in a liquid state. As the simulation proceeds, the system converts portions of the material between the solid and liquid states, as circumstances require.

3.2 Liquid

The liquid state of matter is simulated with a fluid solver. A fluid implicit grid-based approach simulates sand as liquid just below the surface of our simulation. The fluid simulation acts as a buffer between our frozen solid pile of sand and our surface particles. The fluid part of our simulation is the part that gives us most of the continuum behavior that is sought after in sand. As such, our solid mesh is now modeled as cells in our fluid simulation that freeze when the frictional forces are greater than other forces acting upon it. In the presence of greater external forces, these cells convert from a solid state to a fluid state and resume participating in the fluid simulation.

3.3 Gas

Our gaseous state refers to the surface of the granular material. The surface contains individual particles composed of granular assemblies. Aside from texture or describing the surface of the fluid, particles can detach from the simulation to emulate effects such as splashing, spilling, scattering, and settling. This overcomes the limitations of our fluid simulation and gives our sand simulation more believable behavior.

Particles that ride along the surface of the fluid do not collide against one another as collision is computationally expensive and resembles a DEM solution, which is accurate but costly. In the 3-phase solver, particles that glide along the surface do so because of state information sharing. As a particle gets close to the implicit surface of the fluid, it begins to "borrow" characteristics from the fluid closest to the surface.

Particles are increasingly influenced by the velocities of their fluid counterparts the closer they are together. Particles that begin to sink into the fluid either condensate and

join the fluid state or are pushed back along the tangent of the implicit fluid surface. Particles that remain buoyant may clump together and form rivers in an avalanching surface. The extreme overlap of particles may distract the viewer, as particles should not interpenetrate. This is the current state of sand of a hybrid sand simulation. We will propose a few different approaches, and then go on to describe one that works best.

The most straightforward approach to increasing realism is to use discrete element methods for every particle in the simulation. Even with spatial optimization and a reduced number of particles in the simulation. D.E.M can still be too heavy of a solution as we are not interested in individual particles colliding. Our solution seeks to highlight the behaviors of sand as a continuum, which exhibit macro behaviors that are easily recognizable. We do not need to collide particulate assemblies to achieve that level of detail.

A less complex approach could involve a simplified collision detection model with the particles represented as spheres. Collision would be detected using a mark-and-sweep coarse collision detection step, which would identify possible colliding pairs. Those pairs would be pushed apart to avoid collision. Although this approach would reduce the complexity greatly, thousands of particles on the screen would still need to be simulated apart from our fluid simulation. The added complexity of performing a coarse collision detection step on so many particles can be avoided altogether as our desired behaviors mostly come from our fluid simulation. The particles serve to decorate the surface as well as hide the underlying fluid simulation. The solution we propose, and the thrust of the research in this thesis, is to use a reduced fluid solver to overcome the particle interpenetration issue.

Thesis Statement

On the existing hybrid sand solver there exists an issue where particles on the surface of the simulation can clump up because there is no collision detection keep them apart as it is too expensive. We propose to solve this problem by reusing a quadratic solver that is used for density correction on a 3D volume and repurposing it in a 2D context. We do this by recognizing that since particles on a surface are mostly flat, they could be approached as a 2D problem. We build a special surface that we project these particles onto to flatten them out.

We then perform density correction on this surface, which produces a pressure gradient that we then take the reverse gradient. All particles lingering around the surface in 3-space are advected by the gradient field that run along the manifold whose shape runs through 3-space. We build a matrix and constraint vector (that is normally run on a uniform grid) in a non-uniform context, namely surface mesh. We also perform a gradient operation on a non-uniform context, again the surface mesh, when gradients are typically run on uniform grids.

Our major contributions are applying the density correction or quadratic minimizer to a 2D surface that we construct and project particles to, the building of and parameterization of such surface, finding the gradient of resulting pressures, and the trade-off or novel approach in correct the problem of particle clumping without resorting to a heavy handed approach like collision detection.

Chapter 4

Particulate Unilateral Incompressibility

Our approach in preventing gas-state particles from clumping in an unrealistic manner is to apply the density-correction step of sand fluids to the gas-state particles. A major contribution of our work is the restatement of our problem in a lower dimension. Since gas particles only exist at the surface, we take advantage of the 2D manifold property of surfaces, projecting the particles to a 2D volume. This is much more efficient than solving a minimization problem in 3 dimensions, enabling us to address a higher number of particles.

In fluid solvers there are a wide range of trade-offs involving speed and accuracy. For the density correction step, speed is a priority and as such, we can run a solver that estimates the density solution with a few iterations (just enough to push the particles apart) and then stops, providing a rough solution where pressure doesn't necessarily behave in a manner that is physically correct, but where the most apparent interpenetrations are corrected.

As the fluid is a grid our solution can take advantage of existing structures. Running the existing fluid solver would result in surface particles dispersing too rapidly and behaving as a wispy fluid. We don't want the surface to slosh around as it will distract the viewer. Instead, we use a light-weight solution that ignores many behaviors typically associated with fluids.

The fluid will contain a surface of varying height. The difference in height between a fluid cell and its neighbors cannot exceed a certain threshold that is correlated to the angle

of repose. With this guarantee, we construct a manifold. Though the normals of the 2-dimensional grid cells will differ, their underlying pressure solving step will ignore this entirely while the advection step will use the normal as it advects each particle, pushing each particle relative to the fluid surface. See Figure 1 for an outline of our approach.

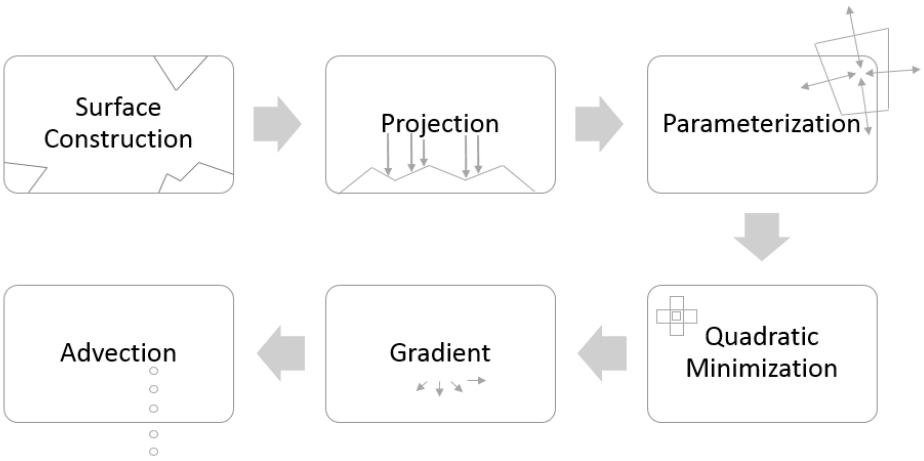


Figure 1: Algorithm Overview

Step 1. Surface construction

Insert primitives between grid points of fluid field.

Add vertices to steep edges.

Flip triangles to normalize surface.

See section 4.3 for details.

Step 2. Project particles

Find closest primitive.

See section 4.4 for details

Step 3. Texturization

Determine adjacent primitives.

Calculate particle density of each primitive.

See section 4.5 for details

Step 4. Density Correction

Build Laplacian matrix.

Run density correction solver.

See section 4.6 for details

Step 5. Gradient

Run pressure difference on each primitive pair.

See section 4.7 for details

Step 6. Advection

Interpolate gradient vectors onto particle.

Update particle position.

See section 4.8 for details

Once 2-dimensional projection vectors are resolved, the original particle will project its velocity vector onto the grid. After the advection step, the particles behave similar to those might have been colliding. The particles do not interpenetrate as frequently as before our method was applied nor act as a singularity giving our sand simulation surface divergence.

Our approach gives the surface of our simulation increased fidelity without the burden of too much computational complexity added by other methods. By introducing a simplified fluid simulator that uses a pressure solver to affect the density correction step, we are able to prevent granular particles from clumping together without introducing the complexity of collision detection and correction. Our method will allow our tri-state granular simulation to look as good as discrete element methods while allowing for faster

simulation times. Our tool will empower artists to iterate on granular simulations quickly giving them greater control over their simulation while reducing time and cost.

4.1 Compression

The issue of clumping is directly caused by an artifact in the method chosen in simulating the liquid layer. We have chosen the fluid-implicit particle method (FLIP) which is a semi-Lagrangian approach. This class of fluid solvers suffers from a type of numerical error called compression [29].

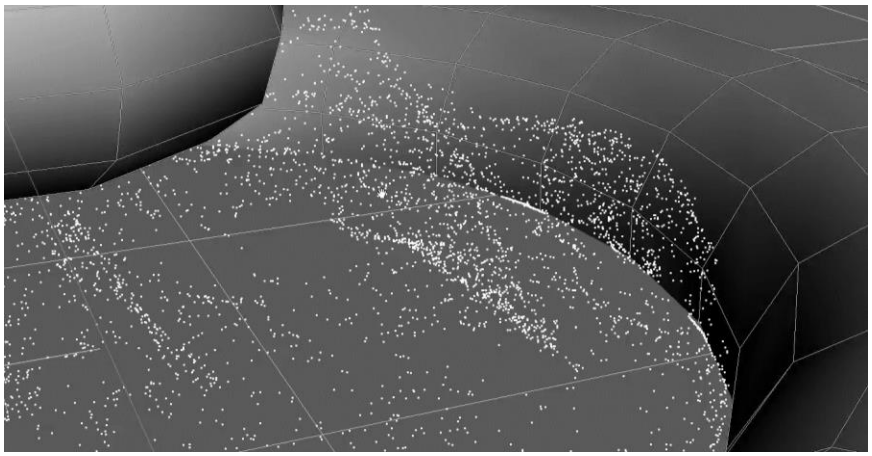


Figure 2 Compression can be seen as clumps of particles form solid lines along the inside of the rim and at points of impact.

Compression is a state when fluid clumps together and resists the pressure solver's correction resulting in a high density area. In fluids, this can result in a loss of volume and produce artifacts in fluid flow. This type of numerical error is acceptable because it is difficult to notice due to the constant motion and chaos in fluid simulations. When two particles are close together, they receive the same advection updates and travel together from the Eulerian solve (see Figure 2). While this works with particles deeply embedded in a fluid simulation, it can have a popping effect on surface particles. Moving particles apart

also does not prevent them from compressing. Density correction prevents clumping and corrects for areas on the surface where particles gather.

4.2 Volumes and Fields

To understand our manifold construction, we introduce our data structure used in our fluid simulation. A voxel field is a three-dimensional structure where values are stored at discreet points along a grid that runs over three axes. Voxel fields are commonly referred to as fields, grids, and volumes. A fog volume is the simplest version where a value of 1 signifies the voxel contains fluid and a value of 0 signifies the absence of fluid (generally considered air but in order to realistically capture interaction air requires a 2-phase fluid simulation). In general, the more (which results in smaller) voxels representing a volume, the more accurate that representation will be at the penalty of memory and computation time. Volumes are generally uniformly spaced with uniform dimensions for simplicity, though they may vary. Non-uniform fields (such as tall cells) [10] must take into account the different voxel areas in computations and spatial derivatives.

Our solution focuses on building a non-uniform surface that lacks the breadth and ease of applying algorithms like finite differencing and minimization to accommodate our 2-dimension surface. We introduce major contributions in building these algorithms for a non-uniform surface of primitives.

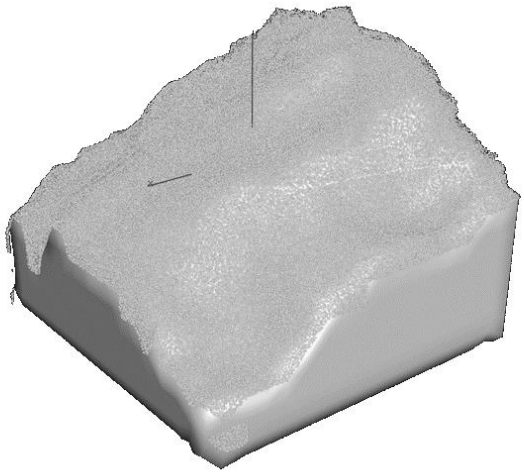


Figure 3: Particles gliding across an implicit volume boundary.

In fog volumes, a surface is built along the border of voxels where 0 meets 1. A limitation of the fog volume is that surfaces must be constrained to a voxel boundary. A signed distance field (SDF) corrects for this by allowing for a floating point value to be stored at the voxel centers which indicate the distance to the closest surface. A negative value may represent the distance inside the surface while positive values indicate the distance above the surface. The surface is implicit as it exists on the 0 boundary, though the boundary value can be set arbitrarily as well. The floating point values allow the surface to be constructed within voxel boundaries allowing for a more accurate fluid boundary (see Figure 3). Signed distance fields are the preferred representation for Euclidian fluid volumes.

Multiple fields can be used to represent different values such as density and velocity. Staggered grids ease the computation of spatial derivatives. Values on a staggered grid are represented on the center of the positive axis faces. This allows for the spatial

derivative to be computed as a closer central difference, (less smoothed out) which lands the result in the center of the voxels of the resulting field, which eliminates the need for an extra trilinear interpolation step. Much research has been put into optimizing grids as a data structure for fluid simulations.

4.3 Surface construction

A surface is constructed on the implicit the fluid signed distance field. Having a surface enables us to enforce a density constraint in only 2 dimensions instead of a 3 dimensional volume. Each voxel is sampled once for the distance to the surface and a point lifted or sunk from the center of the voxel to sit on the surface. More samples may be used to build a more accurate surface at the cost of additional complexity (see Figure 4). The set of points that represent the surface are used to build a mesh. The points act as vertices for the resulting quadrilaterals. The surface can be further refined by looking for disparity in height and adding additional vertices along with their resulting triangles in order to get a more accurate surface.

Inserted vertices that come from the refinement step will fall on an edge and will gravitate towards the points with the greatest difference. The disparity is measured by comparing difference in height among the four shared axel points. Like clothes on a clothing line that is lifted, the point slides closer to the steeper point. If the point slides too close and creates a sliver (a triangle with a low area to edge length), which is degenerate, and the point is removed. Small primitives unnecessarily increase the size of our solver matrix and are avoided. The resulting surface is a combination of triangles and quads.

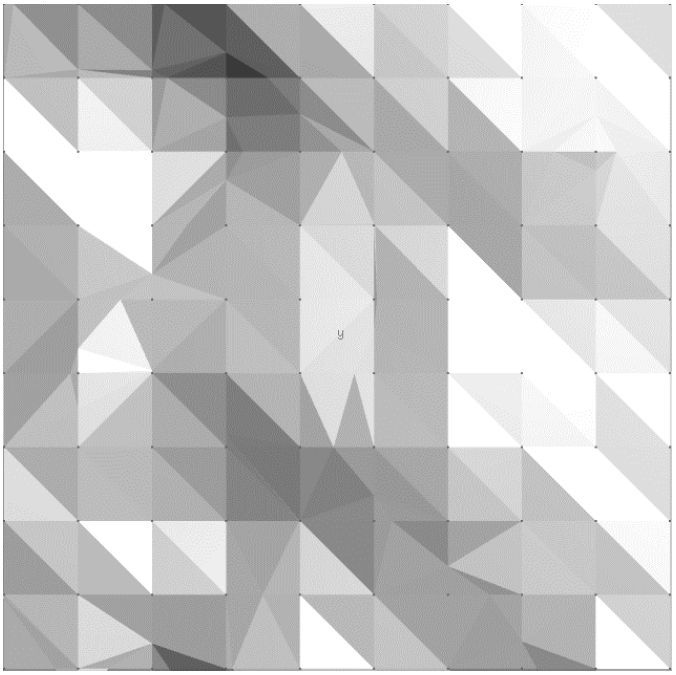


Figure 4: An example of our Surface construction from a signed distance field

Delaunay triangulation can be used to enforce the Delaunay condition minimizing the maximum angles. A simple triangle soup can be built by of randomly adding a point from the point set, forming a triangle and flipping if needed. Though the surface used does not need enforce the Delaunay condition. A balanced algorithm for triangulation is important to keep the surface as uniform as possible as it gives the solver predictable areas for pressure to escape to. Delaunay is often the go to algorithm with triangularization. The surface, however, may also be entirely comprised of triangles by using Catmull-Clark subdivision if one so chooses. A mixture of polys and triangles will work equally well for our purposes as our algorithm generalizes to any polygon.

The surface comprises the boundary of a 3-dimensional volume. As we are only interested in the top surface, a group of polygons is formed by calculating each face normal

and filtering out every face that differs from the angle pointing directly upwards by a maximum of 85 degrees arbitrarily chosen to be near vertical. This constraint means that the only faces to remain are those that face relatively upwards. Those faces are the most likely to support the bulk of the gas particles. This filtering is a trade-off that allows for a smaller problem set.

4.4 Projection

A trade-off between realism and complexity is made with the projection from 3-dimensions to 2-dimensions. The difference in speed in each frame is justifies the error in accuracy introduced with projection. As a granular simulation will mostly be viewed from above, subtle difference in particle height go unnoticed. Taking advantage of this fact we approach the problem of density correction as a 2-dimensional problem and project particles that are close to the fluid surface onto the newly created surface.

The particles that represent the gaseous state of the granular simulation are located on the fluid surface and are allowed to slide along that surface as necessary. As gravity overcomes the hydrophobic properties of the surface, the particles coalesce and join the fluid. This results in the majority of the gas particles floating just above the fluid surface. As particles exit this space, they are either ignored if they travel above or converted below into the fluid state. In order to spread these particles apart along the surface, they are projected onto the newly built surface.

Gas particles can be projected onto the surface by using a minimum distance criteria. Points along the surface and particles can be paired by using a divide and conquer closest point-pair algorithm. A more straightforward approach involves gas particles using

ray-triangle intersection with a downward normal (along gravity). The ray-triangle intersection can give the point of collision in Barycentric coordinates and ray-polygon intersection in parametric coordinates.

$$\rho_f = \frac{\sum_{p=0}^{n} \mathrm{f}_p}{a} \tag{1}$$

As each gas particle projects to a single polygon, the relationship becomes a one-to-many relationship where each polygonal face, f, can have any number of gas particles project to it. The concentration of particles on each polygon normalized by the area, a, of the polygon determines its density, ρ (1). The density of every polygon is needed to enforce the global density constraint in the density correction step.

4.5 Texture Parameterization

The construction of a fluid surface and filtering out faces by their normals allows for the planning of a 2-dimensional manifold that can host a lightweight solution to the problem of interpenetrating particles. Building the surface as a manifold abstracts away 3-dimensional spatial details in favor of relative position in a 2-dimensional context.

Manifolds are offered referred to as textures and are parameterized along two dimensions commonly labeled U and V to avoid confusion with Euclidian spaces. Mapping textures to meshes or subdivision surfaces is a common problem in animation. The ideal parameterization of this manifold provides a mapping from an individual face to all neighboring faces that share an edge (see Figure 5). A per-face UV parameterization with

adjacency data called Ptex has been developed by Disney Animation for efficient and general filtering across textures.

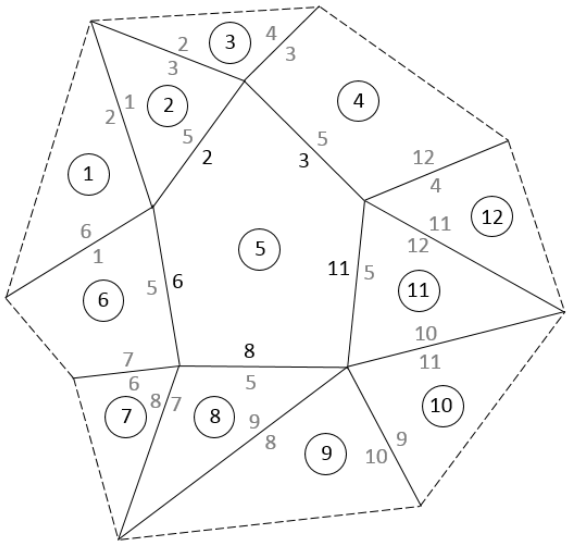


Figure 5: Ptex parameterization of neighboring faces establishing adjacency.

Ptex is a per-face parameterization where U and V texture coordinates are local to the face. Along with individual U and V coordinates, the Ptex texture format also encodes neighboring (faces that share an edge) faces to allow for easy convolution and compression. A neighbor or adjacent primitive is determined by a shared edge. There is no need to parameterize the surface globally.

4.6 Quadratic Minimization

Sand is unilaterally incompressible. This means that while sand cannot be compressed beyond a certain threshold, it also does not force the system to become divergence free. Our density correction step is a constraint that enforces unilateral incompressibility. Though it can be compressed beyond a certain threshold, it may be free to disperse

indefinitely, or only experience compression in only one direction. Sand differs from water in that pressure will not infinitely increase given an infinite force. We determine a minimum density constraint that each face is bound to in order to avoid excessive particle overlap as demonstrated in Narain et al [26]. Narain et. al introduces a density constraint to correct for overly dense conditions on their granular fluid simulation. A unilateral constraint means that pressure is only applied in areas where density has hit a threshold. Areas below this threshold are largely unaffected [27]. The constraint is satisfied by moving particles away from cells that violate it. The minimum amount of movement needed to satisfy the constraint is phrased as a minimization problem (see 2).

$$F = \frac{1}{2} p^T A_1 p + b_1^T p \qquad (2)$$

We adapt this formulation to our surface particle problem. Because of quadratic terms, we solve the minimization problem using a custom quadratic solver built in our research lab and included in previous work [3]. The matrix A is a Laplacian sparse matrix describing the density of faces and their relationship with their neighbors (see Figure 6). The vector p is our pressures that we are trying to solve for and must be positive. The vector b represents our density constraint.

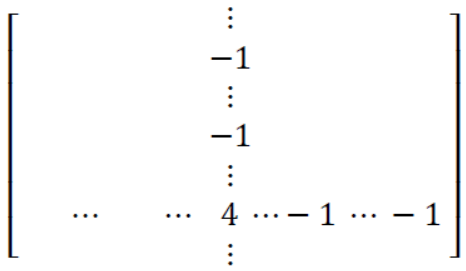


Figure 6: Linear Differencing kernel representing a single row/column entry

Each primitive or cell is assigned an index. For every index of that primitive, there is a corresponding row and a corresponding column. For example, primitive number 5 will insert its values into row 5 and column 5. A surface comprised of 1000 faces, for example, would result in a square matrix of 1000 rows and 1000 columns. Like a Laplacian kernel, the Laplacian sparse matrix behaves like a second order operator. In the position where the corresponding row and column index intersect, the kernel value of the primitive at that index is inserted. For each neighbor n at index j, a value of -1 also is inserted corresponding to their position the row and column of the matrix.

$$A_{i,i} = \frac{\rho_i}{\rho_{max} a^2} |n| \,, \quad A_{i,j} = A_{j,i} = -\frac{1}{a_j} \tag{3}$$

This results in an upper triangular, symmetric, and sparse matrix. The kernel value inserted is the density, or particles per area divided by the maximum allowed particles. Our vector b contains length |faces| with volume fractions $1 - \frac{\rho}{\rho_{max}}$ for each face in the index corresponding to the primitive index. Our solution vector p contains pressures for each face (3).

4.7 Gradient

Particles near the implicit surface are advected by taking the reversed gradient of the surface and moving particles along the resulting vector field. The quadratic minimizer leaves a pressure, $\boldsymbol{\Psi}$, which is associated to each primitive face. Instead of having a normal voxel grid of pressures, however, we have a non-uniform surface that has been discretized by constructing triangles and quadrilaterals along the implicit boundary of the hybrid simulation.

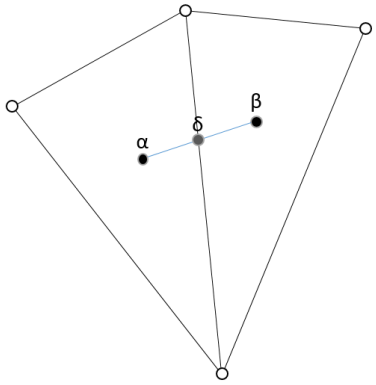


Figure 7: Central differencing on a non-uniform manifold.

We implemented a modified central differencing approach to adapt the gradient to a non-uniform surface. The resulting vector does not have to be constrained to the surface (see Figure 7). With our geometry in ptex format, we have access to all adjacent primitives for any given primitive. Central differencing generates a vector with each pair. The base of the vector is the midpoint between two primitive centroids possibly generated outside the plane of either primitive. The direction of the vector points to the centroid of the primitive that contains a lower pressure value (4). Vectors may be degenerate in the case of equal pressure values. α, β correspond to the centroid position vectors. $\boldsymbol{\Psi}_\alpha, \boldsymbol{\Psi}_\beta$ are the

pressures of the corresponding primitives. δ is the resulting gradient vector with position at 0 and direction at norm.

$$\delta_0 = \frac{\alpha+\beta}{2}, \quad \delta_{\hat{u}} = \frac{\max[\boldsymbol{\Psi}_\alpha, \boldsymbol{\Psi}_\beta]}{||\alpha-\beta||} \tag{4}$$

The vectors are normalized then scaled as pressures take into account differences in area and density and are expressed in direction. The resulting vector field is also non-uniform. Central differencing is ideal for adjacent primitives as forward or backward differencing methods require a fully parameterized surface which is unnecessary. Accuracy in producing the gradient is also sufficient using central differencing.

4.8 Advection

Each particle is advected along the resulting gradient field. The field does not apply a force as forces can be problematic as they can cancel out other forces in the particles. Instead, the granules are instantly displaced in time along the field. The displacement is scaled by the simulation time step to prevent unnatural movement or introducing energy into the system.

$$\sum_{p=0}^{n} \frac{force_p}{(x-x_p)^2+(y-y_p)^2+(z-z_p)^2} \tag{5}$$

Particles receive interpolated direction from the field using a weighted proximity technique. A meatball [30] kernel determines proximity using a distance threshold among the point and its nearby vectors (5). Metaballs are ideal for setting up fields of influence. The kernel gives a higher weight to vectors that are closer. This results in points' positions being largely determined by their local neighborhood while also being influenced by wider

trends in the system. After particles have been moved along the gradient field, they continue to run through the hybrid simulation, reacting to external forces, coalescing, and separating as normal.

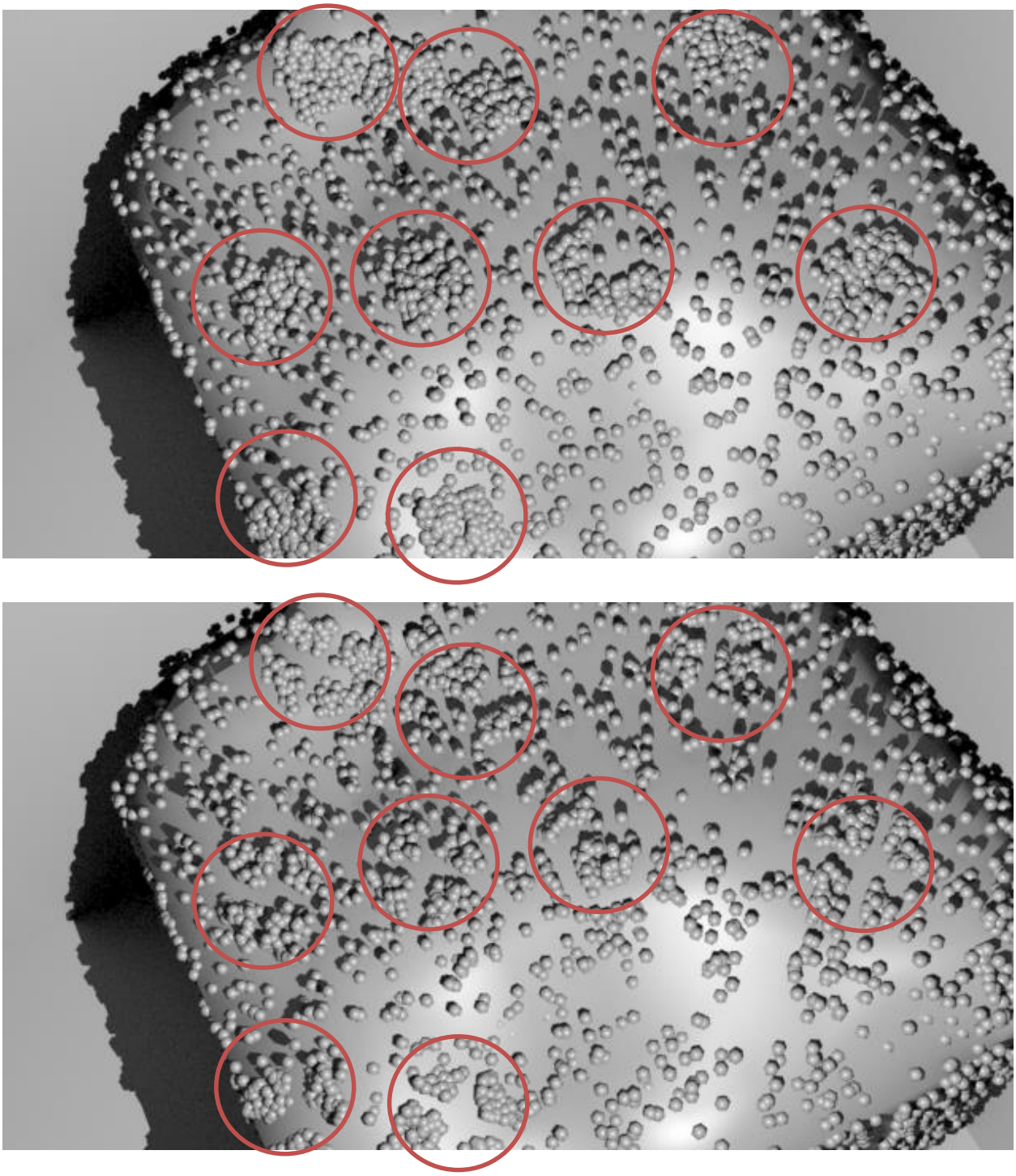
Chapter 5

Results

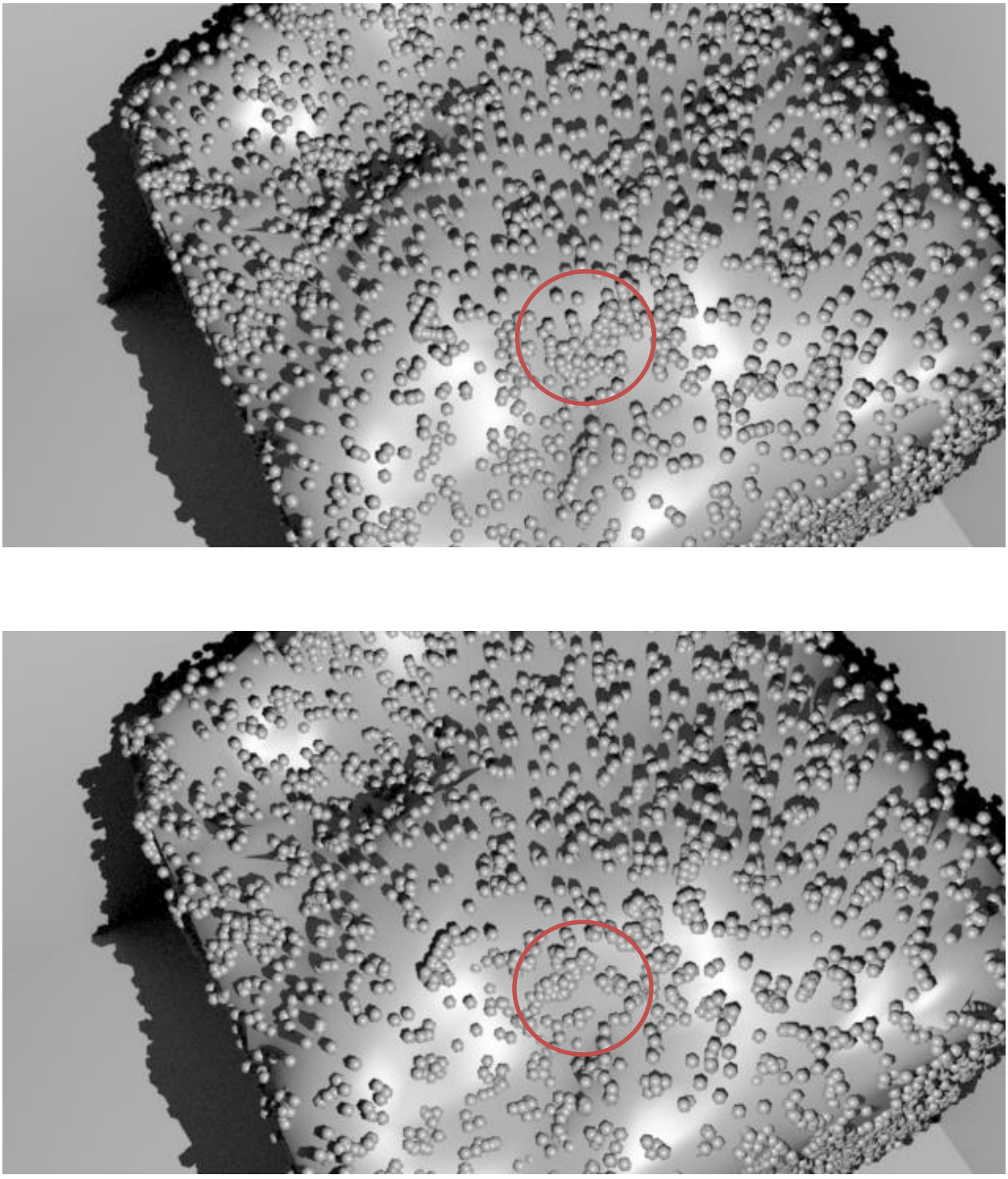
We set up a signed distance field volume that was initially 10x10x10. The volume has a random function that generates interior and exterior volume for our algorithm to run on. We populated the surface of the volume with 10,000 particles. Particle distribution is biased to simulate the condition of clumping on a full solver (see Figure 8).

Results of the simulation are run a small grid as well as a higher grid. To render each frame takes < 1 second on a 4.0 Ghz AMD 8 core machine with 16 Gb of memory on a virtual machine allotted 50% of those resources. The algorithm is not intended to be real time. The fluid step on a sand solver can typically take 30 seconds to 3 minutes on a similar machine and similar grid. On a 1000x1000x1000 volume the simulation takes 3-12 minutes depending on the initial conditions of the solver. Solving over a 3d volume would take hours.

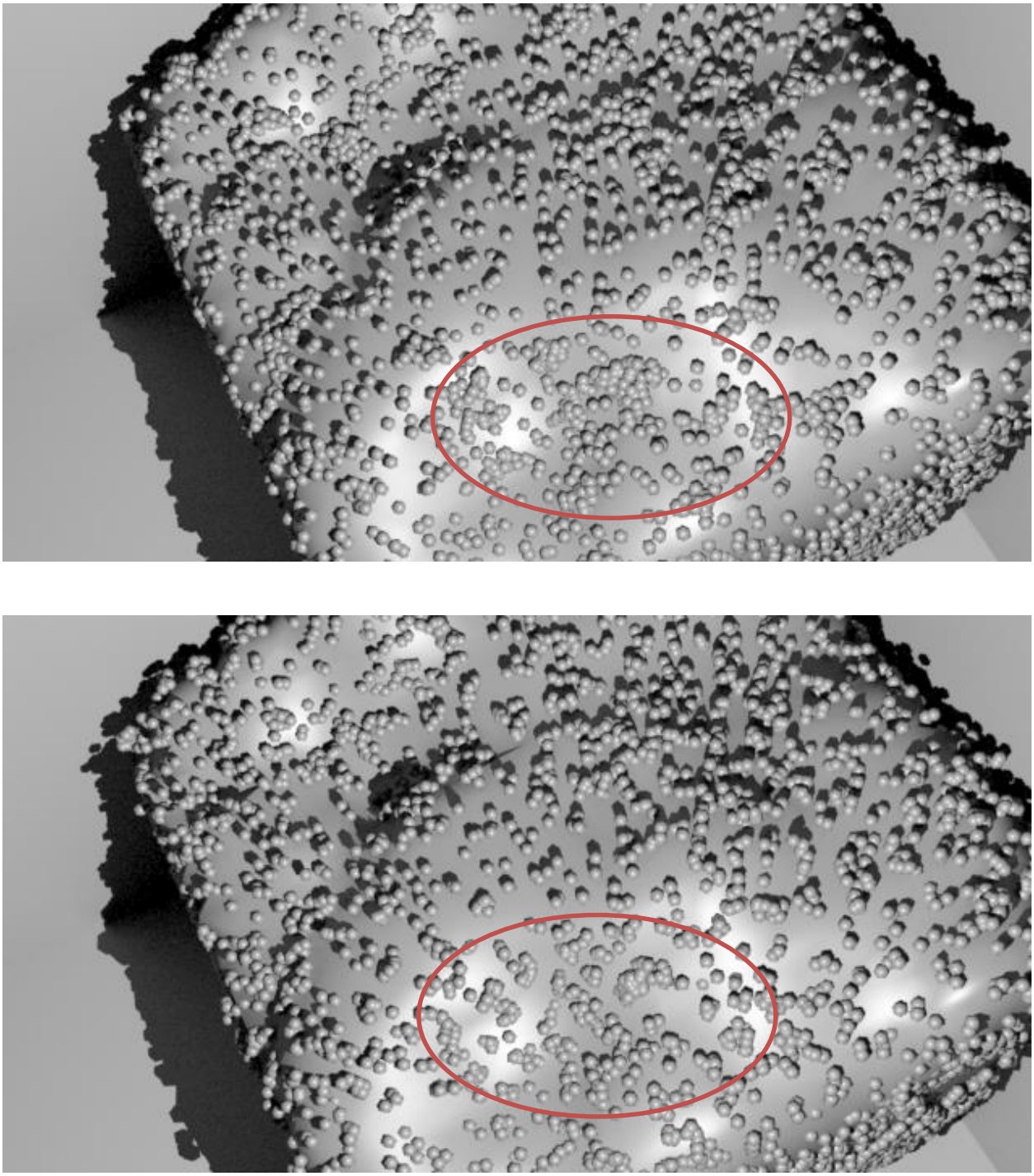
The first set of results shows a volume with particles forming in 9 individual clusters. In each of these clusters, the algorithm detected violation of the density constraint and flagged the primitive with a high pressure. Lower pressure primitives surrounding pull the particles apart. The next set of results were run on a higher voxel grid with more primitives creating the surface. A cluster of particles is detected and dispersed. The third set of results is run at the same resolution with different random initial conditions and shows a more broad resolution of the particles.



Results 1: Single pass with low resolution mesh (10^{30}) shows overpopulated quads pulling particles away from the center. 10,000 particles on surface. Constraint of 20 particles per face.



Results 2: High resolution mesh (1000^3) shows overpopulated quad pulling particles away from the center. 10,000 particles on surface. Constraint of 20 particles per face. Particles that do not exceed the threshold do not get pushed apart.



Results 3: Another high resolution mesh (1000^3) with a different surface. Constraint of 20 particles per face. Dispersion of particles in overly dense areas.

Chapter 6

Discussion and Future Work

Our solution performs well at breaking apart clumps that upset our predefined density constraint. In every case where particles violate this constraint, they are globally moved into lower pressure areas to obey the constraint.

One shortcoming of our algorithm is that our density constraint has to be specified ahead of time. Future could involve automatically finding a threshold based on the distribution of the particles at the start of the frame and adjusting to enforce a constraint on a certain deviation of that density. Another issue is when the constraint is set too high and the system is too laden with particles to be able to effectively find a solution where the constraint isn't violated becomes impossible. This is only avoided through trial and error.

Our formation of the problem was from the primitive or cell point of view. We just have easily formulated the problem from a vertex point of view, which would allow us to perform operations like the gradient as a vertex shader potentially speeding up the process and giving us access to the graphics card for parallel computation.

The construction of the surface from the implicit voxel boundary alternatively could be dependent upon the density of the particles around it. More triangles and quadrilaterals under particle-dense areas would allow for more gradient vectors resulting in finer movement. More computation would be spent on problem areas as opposed to a mostly uniform advection solution.

Another possibility for dispersing patches is to apply more than one pass of our algorithm relaxing the constraint on each pass in order to get additional smoothness. The motivated researcher may even apply machine learning to reach a solution that optimizes for both speed and lower density given the inputs of number of passes, density constraint, and various scalar values found throughout the equations like advection intensity.

Chapter 7

Conclusion

We have demonstrated a superior method for handling surface particles on a hybrid simulation. Our method solves the issue of particles getting too close together and clumping unnaturally, distracting from the simulation. We have solved this issue without adding a computational bottleneck and meeting the performance goals of the hybrid solver.

Our contributions include a unique rephrasing of the problem of running our density correction step from a 3d volume to a 2d manifold making a very important trade-off in having a faster density correction step while relenting only slightly on the visual acuity of the algorithm. As a result, we have extended the 3-phrase hybrid simulation to fix an artifact on the surface where particles clumped.

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